Schneller als das Licht?

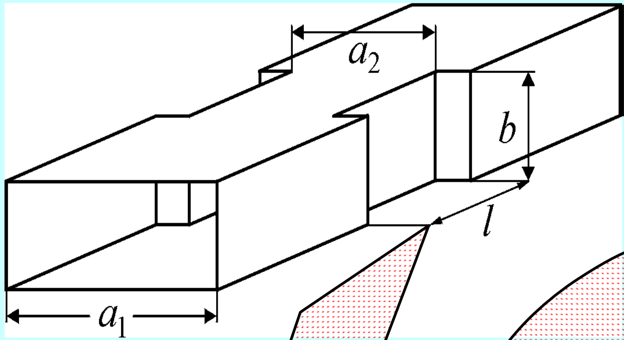
Zur Geschwindigkeit der Übertragung elektromagnetischer Wellen

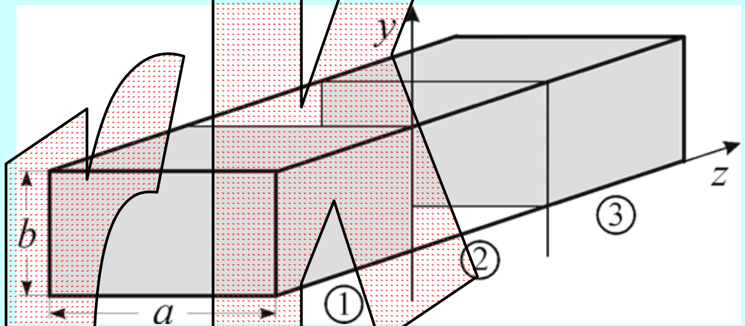
Prof. Dr.-Ing. Klaus W. Kark

FH Ravensburg-Weingarten

Superluminale Strukturen

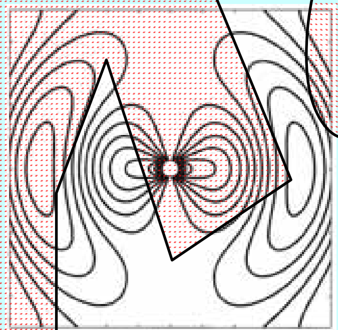
Cutoff-Wellenleiter





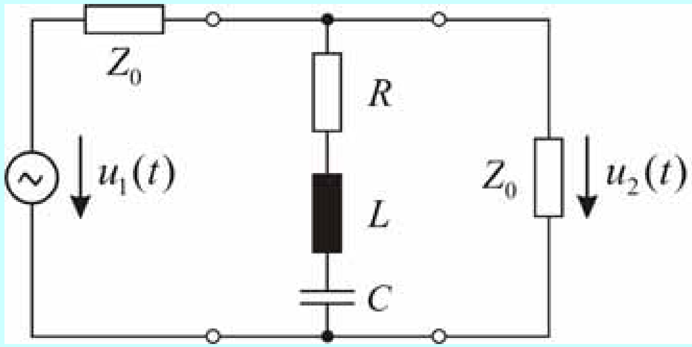
Antennen-Nahfeld

(z.B. Hertzscher Dipol)

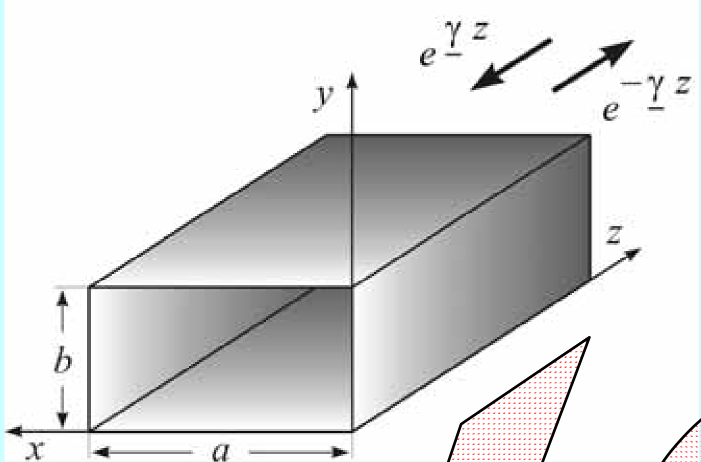


RLC-Netzwerke

(z.B. Bandsperre)



Geschwindigkeitsdefinitionen I



Ausbreitungskonstante $\underline{\gamma} = \alpha + j\,\beta$

Dämpfungskonstante $\alpha(\omega)$

Phasenkonstante $\beta(\omega) = \dfrac{2\pi}{\lambda_L(\omega)}$

Phasengeschwindigkeit
(d´Alembert 18. Jh.)

$$\boxed{\mathrm{v}_p(\omega) = \frac{\omega}{\beta} = \lambda_L\, f}$$

Gruppengeschwindigkeit
(Hamilton 1839, Stokes 1876,
Rayleigh 1894)

$$\boxed{\mathrm{v}_g(\omega) = \frac{d\omega}{d\beta} = \frac{1}{d\beta/d\omega}}$$

normale Dispersion : $\mathbf{v}_g < \mathbf{v}_p$**, anomale Dispersion :** $\mathbf{v}_g > \mathbf{v}_p$

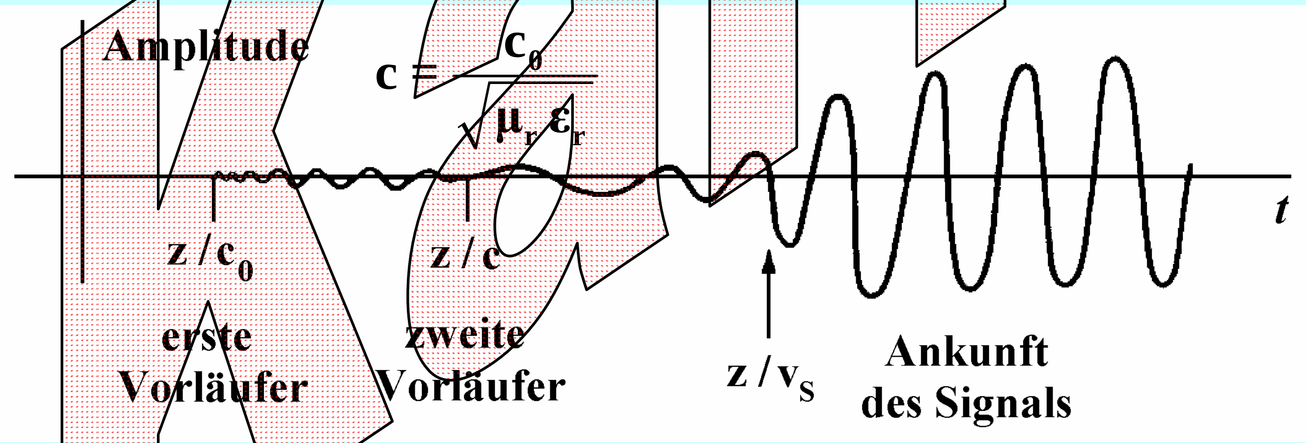
Geschwindigkeitsdefinitionen II

Frontgeschwindigkeit

$$v_F = \lim_{\omega \to \infty} v_p(\omega) = \lambda_0 \, f = c_0$$

Energiegeschwindigkeit

$$v_E(\omega) = \frac{\left|\vec{S}_R\right|}{w} = \frac{\frac{1}{2}\left|\mathrm{Re}\{\underline{\vec{E}} \times \underline{\vec{H}}^*\}\right|}{\frac{\varepsilon}{4}\left|\underline{\vec{E}}\right|^2 + \frac{\mu}{4}\left|\underline{\vec{H}}\right|^2} \le c_0$$

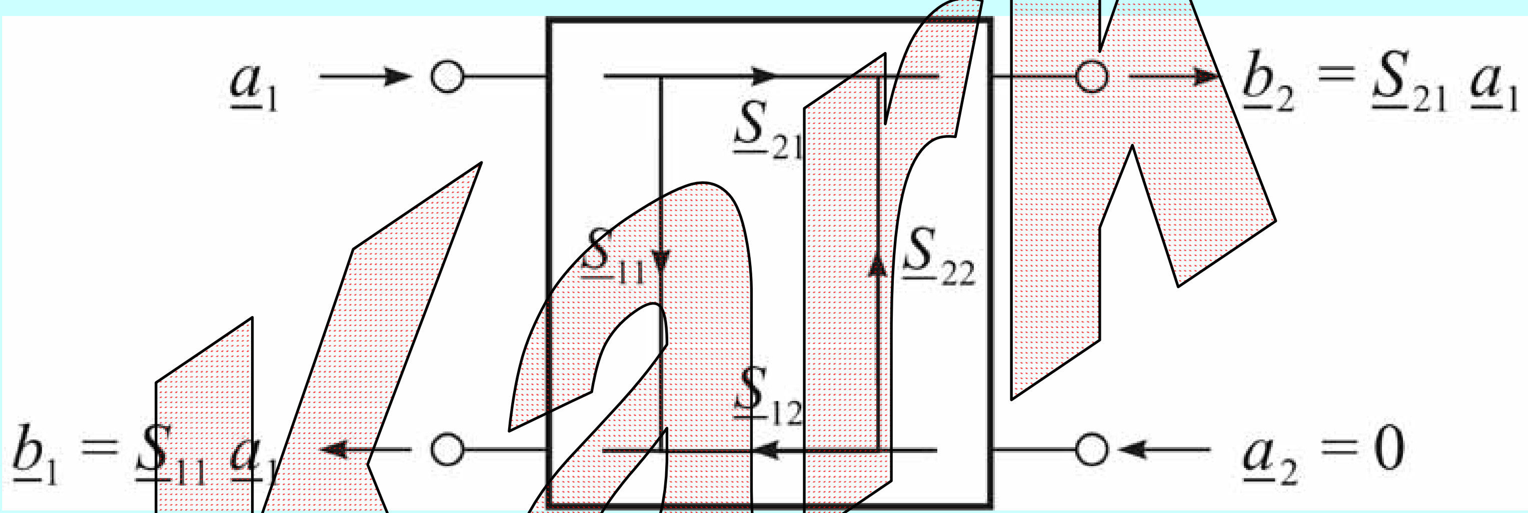


Sommerfeld Brillouin (1914)

Stratton (1941)

Laufzeitdefinitionen I

Übertragungsfunktion $\underline{H}(\omega) = \left|\underline{H}(\omega)\right| e^{j\,\Phi(\omega)} = \underline{S}_{21}(\omega)$



Phasenlaufzeit und Gruppenlaufzeit (Feldtkeller 1935)

$$t_p(\omega) = -\frac{\Phi(\omega)}{\omega}$$

$$t_g(\omega) = -\frac{d\,\Phi(\omega)}{d\,\omega}$$

Laufzeitdefinitionen II

Impulsantwort

$h(t < 0) = 0$ bei Kausalität

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \underline{H}(\omega)\, e^{j\omega t}\, d\omega$$

Schwerpunktlaufzeit

(Marko 1956)

$$t_s = \frac{\int_0^\infty t\, h(t)\, dt}{\int_0^\infty h(t)\, dt} = t_g(\omega = 0)$$

Impulslaufzeit

(Morgenstern 1971)

$$t_i = \frac{\int_0^\infty t\, h^2(t)\, dt}{\int_0^\infty h^2(t)\, dt} = \frac{\int_0^\infty t_g(\omega)\, \left|\underline{H}(\omega)\right|^2 d\omega}{\int_0^\infty \left|\underline{H}(\omega)\right|^2 d\omega}$$

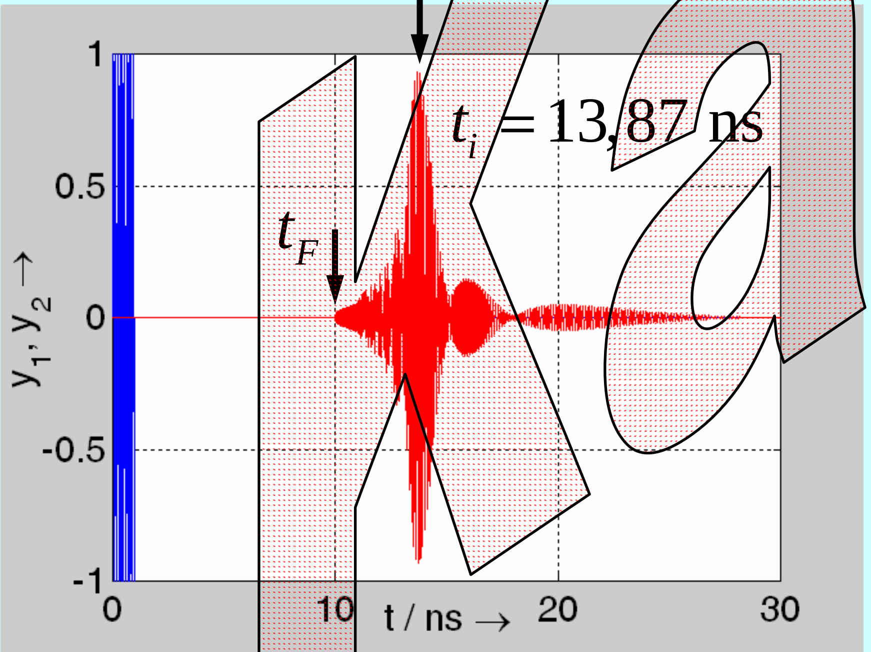
Dispersion in homogenen Hohlleitern

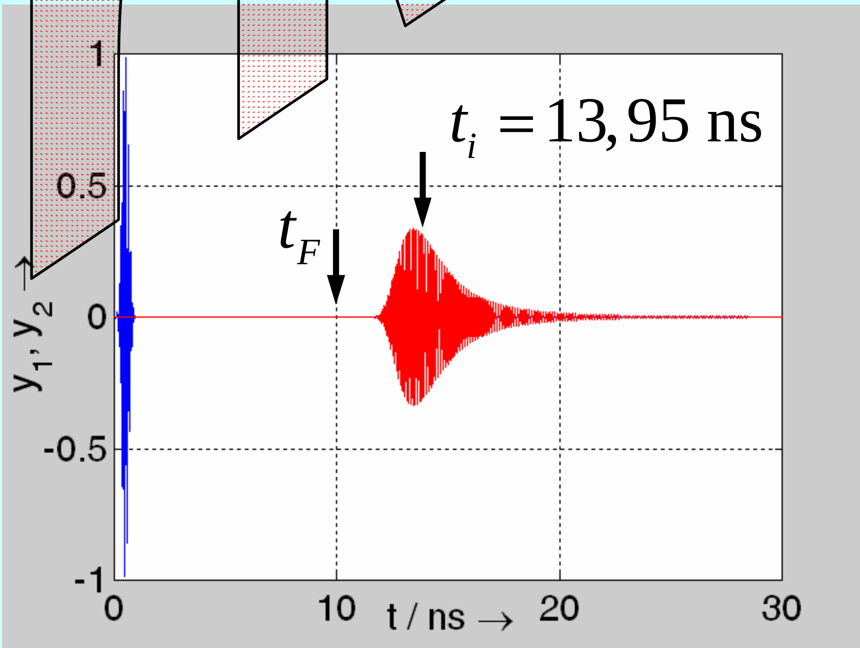
Al-Rechteckhohlleiter (luftgefüllt, reflexionsfrei abgeschlossen)

$$a = 2{,}286\ \text{cm} \qquad b = 1{,}016\ \text{cm} \qquad l = 2{,}998\ \text{m}$$

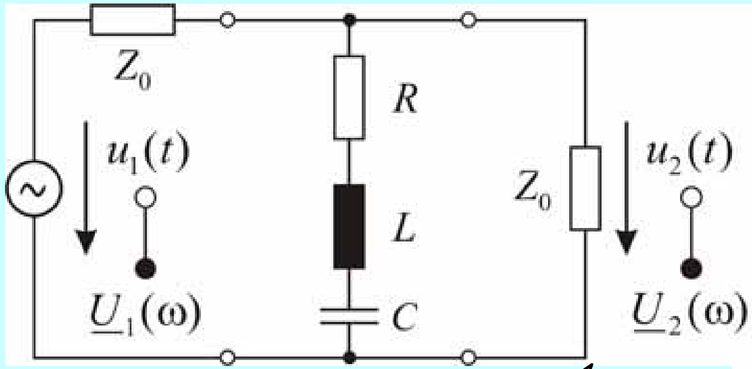
$$\text{Frontlaufzeit: } t_F = l/c_0 = 10\ \text{ns}$$

Rechteck - und Gaußimpuls mit $\tau = 1$ ns und $f_0 = 10$ GHz (X - Band)





RLC-Filterschaltung (Bandsperre)

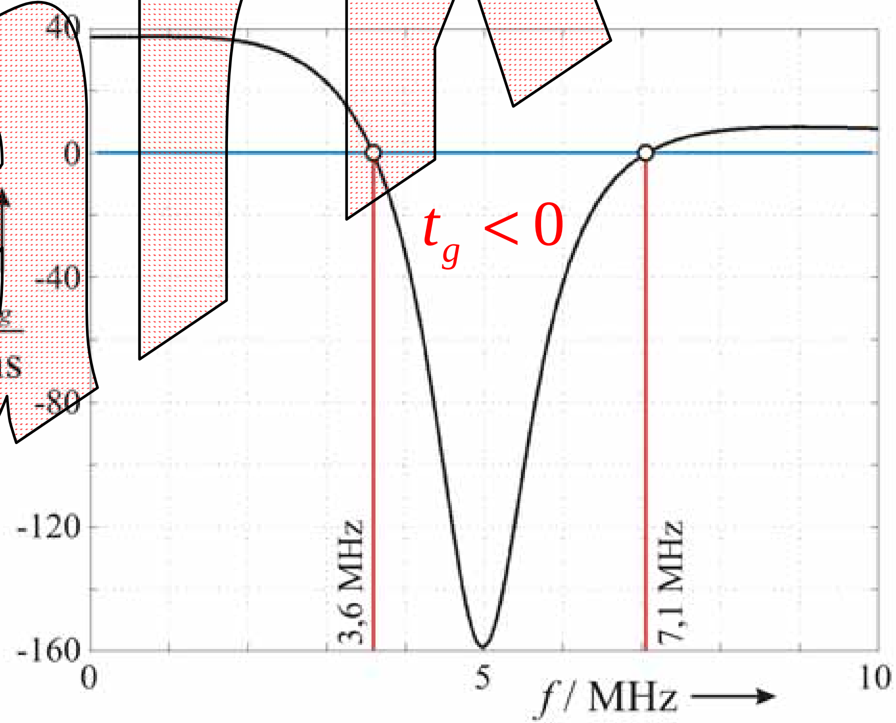


$$Z_0 = 75\ \Omega, \quad R = 10\ \Omega$$

$$\left.\begin{array}{l} L = 1\ \mu\text{H} \\ C = 1\ \text{nF} \end{array}\right\} f_0 = 5{,}033\ \text{MHz}$$

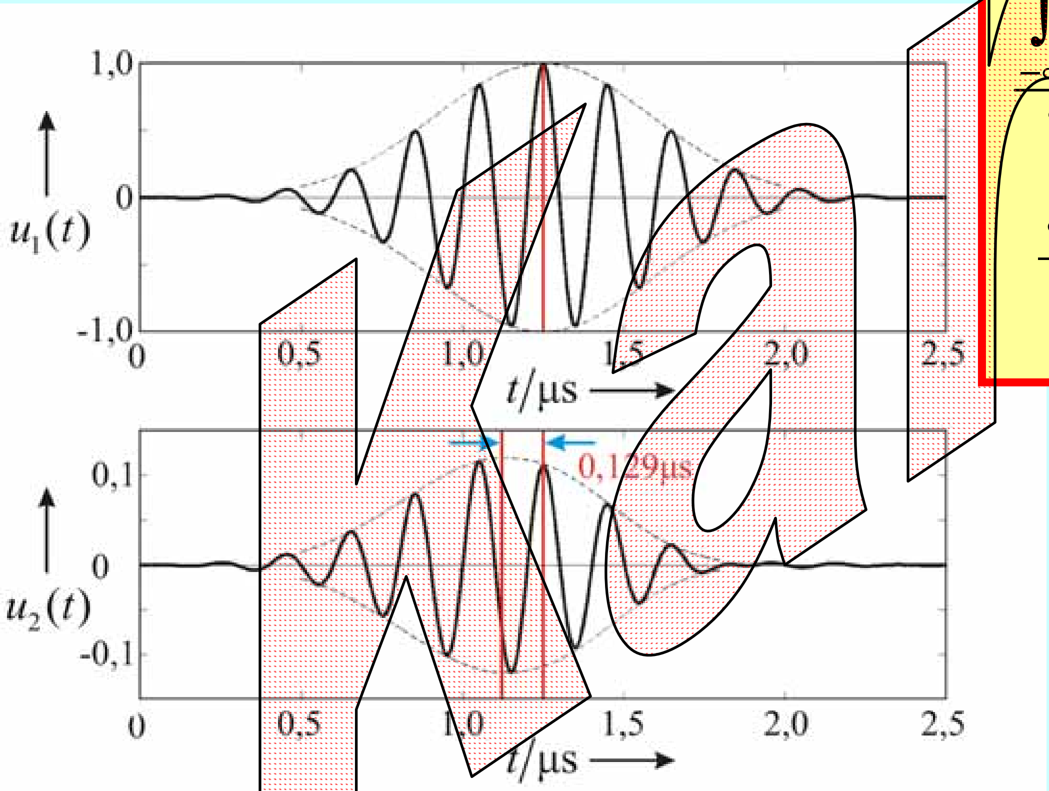
$$\frac{\underline{U}_2(\omega)}{\underline{U}_1(\omega)} = \left|\underline{H}(\omega)\right| e^{j\Phi(\omega)} = \frac{0{,}5\cdot\left(\omega^2 - j\omega\,\dfrac{R}{L} - \dfrac{1}{LC}\right)}{\omega^2 - j\omega\,\dfrac{R+Z_0/2}{L} - \dfrac{1}{LC}}$$

$$t_g(\omega) = -\frac{d\,\Phi(\omega)}{d\omega}$$



Bewegung des Energieschwerpunktes

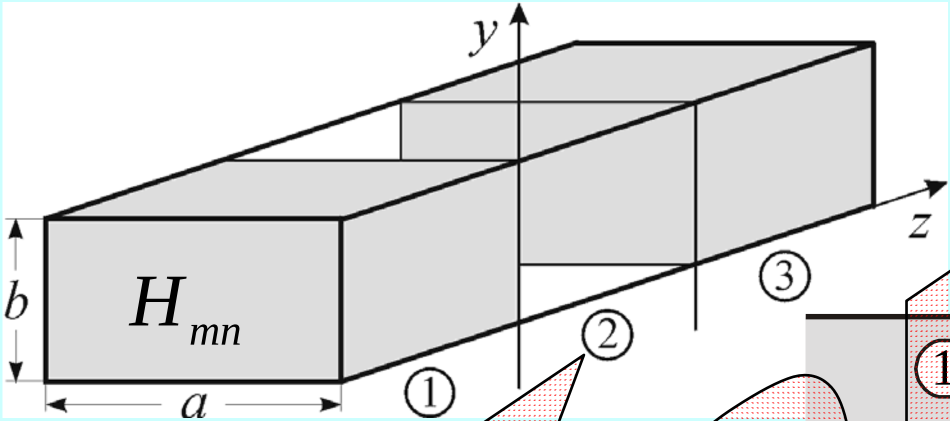
modulierter Gaußimpuls bei $f_0 = 5\,\mathrm{MHz}$,
60-dB-Breite von 2,5 μs



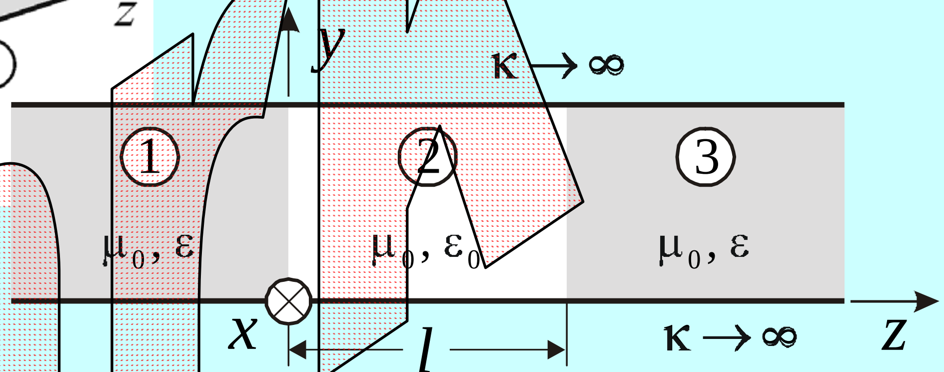
$$\frac{\int_{-\infty}^{\infty} t\, u_2^2(t)\, dt}{\int_{-\infty}^{\infty} u_2^2(t)\, dt} - \frac{\int_{-\infty}^{\infty} t\, u_1^2(t)\, dt}{\int_{-\infty}^{\infty} u_1^2(t)\, dt} =$$
$$= -0{,}129\,\mu\mathrm{s}$$

Impulsverzerrung mit Bildung eines früheren Schwerpunktes, aber:
kausale Übertragung
Rupprecht (1961)

Hohlleiter mit unterbrochenem Stoffeinsatz



$$\omega_c = c_0 \sqrt{(m\pi/a)^2 + (n\pi/b)^2}$$



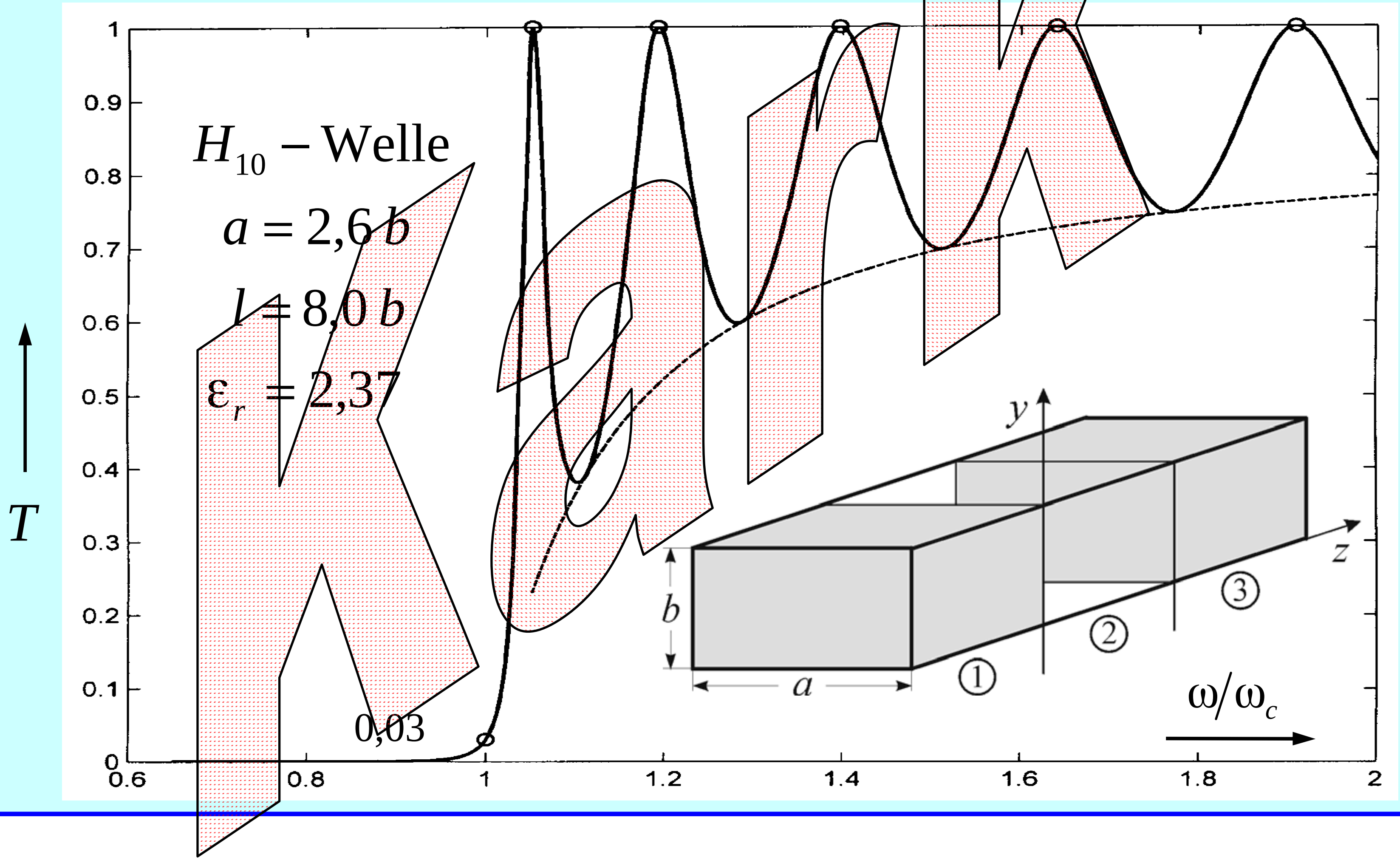
Ausbreitungskonstanten:

$$\underline{\gamma}_1 = \underline{\gamma}_3 = j\,\beta_1 = j\sqrt{\varepsilon_r\,\omega^2 - \omega_c^2}\Big/c_0 \quad \text{für } \omega \geq \omega_c\Big/\sqrt{\varepsilon_r}$$

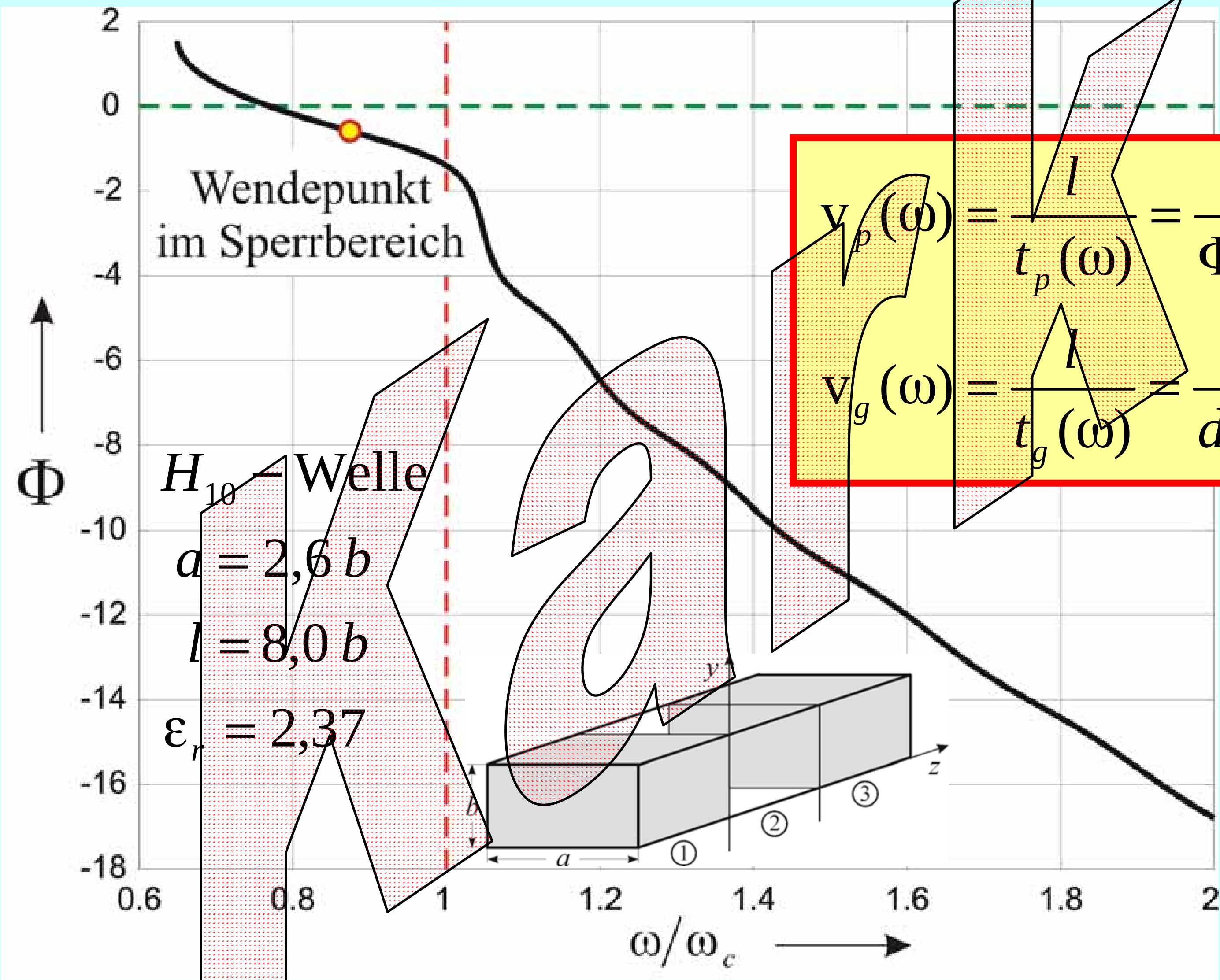
$$\underline{\gamma}_2 = \begin{cases} \alpha_2 = \sqrt{\omega_c^2 - \omega^2}\Big/c_0 & \text{für } \omega \leq \omega_c \\ j\,\beta_2 = j\sqrt{\omega^2 - \omega_c^2}\Big/c_0 & \text{für } \omega \geq \omega_c \end{cases}$$

Die transmittierte Energie $T = \left|\underline{S}_{21}\right|^2$

Sperrbereich $\frac{\omega_c}{\sqrt{\varepsilon_r}} \le \omega \le \omega_c$ Durchlassbereich $\omega \ge \omega_c$



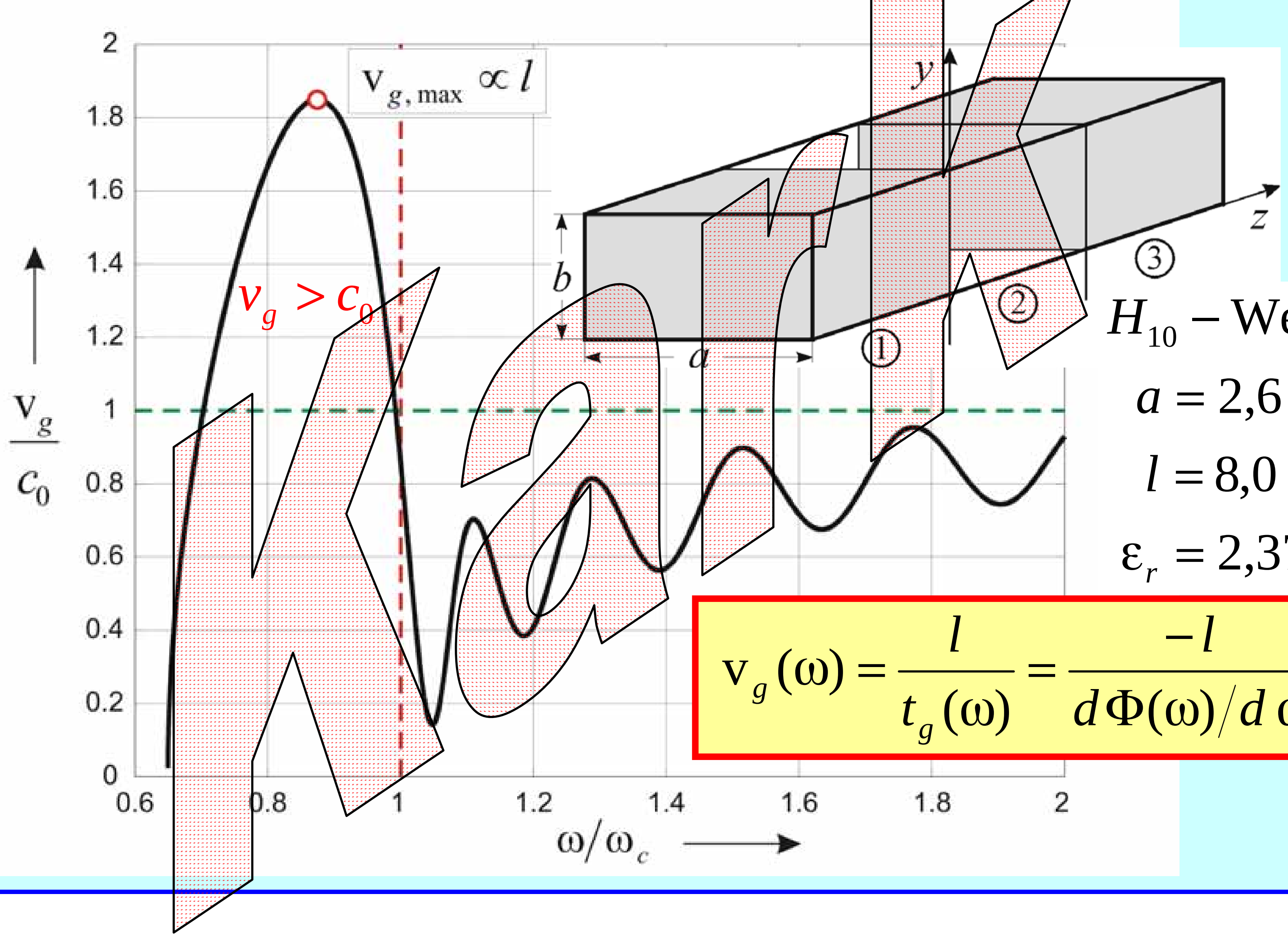
Die Phase Φ der transmittierten Welle



$$v_p(\omega) = \frac{l}{t_p(\omega)} = \frac{-l}{\Phi(\omega)/\omega}$$

$$v_g(\omega) = \frac{l}{t_g(\omega)} = \frac{-l}{d\Phi(\omega)/d\omega}$$

Die Gruppengeschwindigkeit



H_{10} – Welle

$a = 2{,}6\, b$

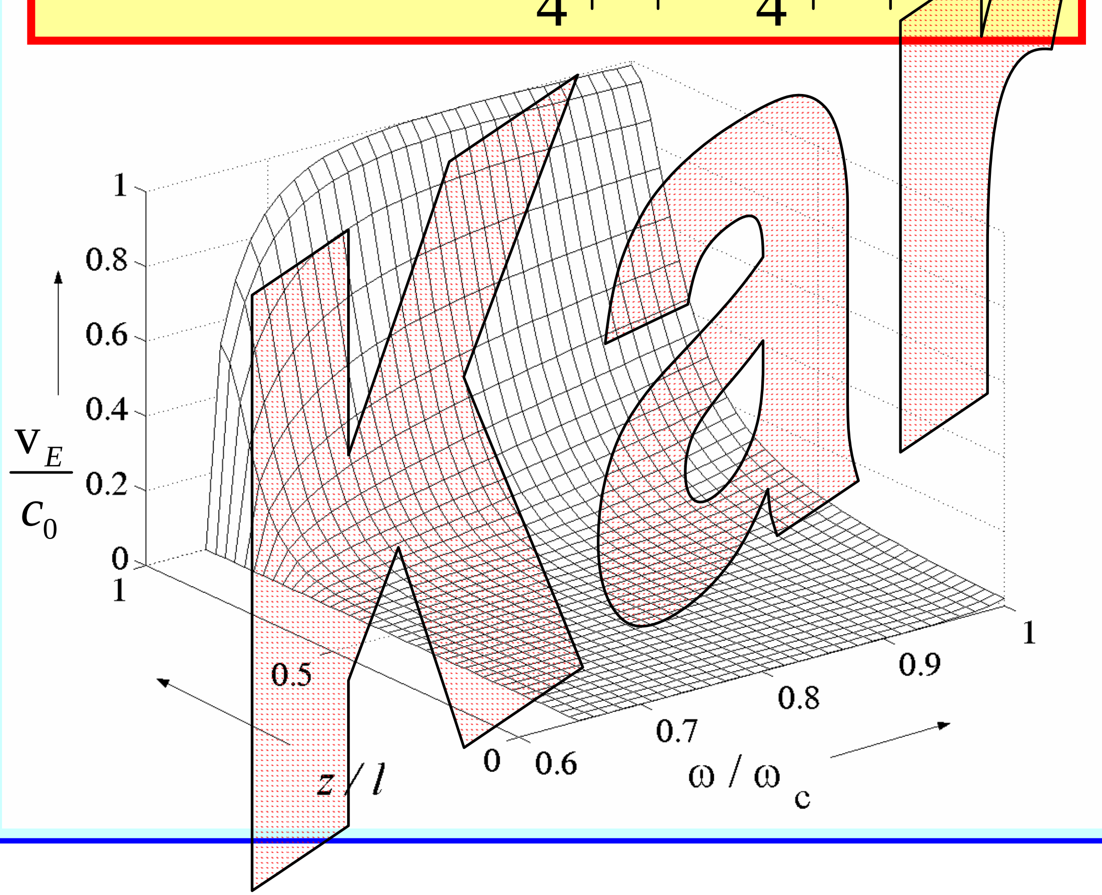
$l = 8{,}0\, b$

$\varepsilon_r = 2{,}37$

$$v_g(\omega) = \frac{l}{t_g(\omega)} = \frac{-l}{d\Phi(\omega)/d\omega}$$

Die Energiegeschwindigkeit

$$\mathrm{v}_E\left(\omega, z, x=\frac{a}{2}\right)=\frac{\frac{1}{2}\left|\mathrm{Re}\{\underline{\vec{E}}\times\underline{\vec{H}}^*\}\right|}{\frac{\varepsilon}{4}\left|\underline{\vec{E}}\right|^2+\frac{\mu}{4}\left|\underline{\vec{H}}\right|^2}\le c_0$$



$a = 2{,}286$ cm

$b = 1{,}016$ cm

$l = 5$ cm

$\varepsilon_r = 2{,}56$

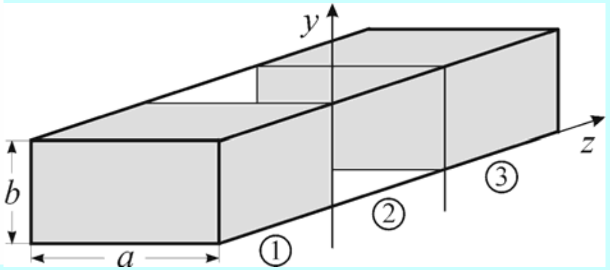
$f_{c,1} = 4{,}098$ GHz

$f_{c,2} = 6{,}557$ GHz

$\cos^2 -$ Impuls

$f_0 = 6\ \mathrm{GHz} = 0{,}915\ f_{c,2}$

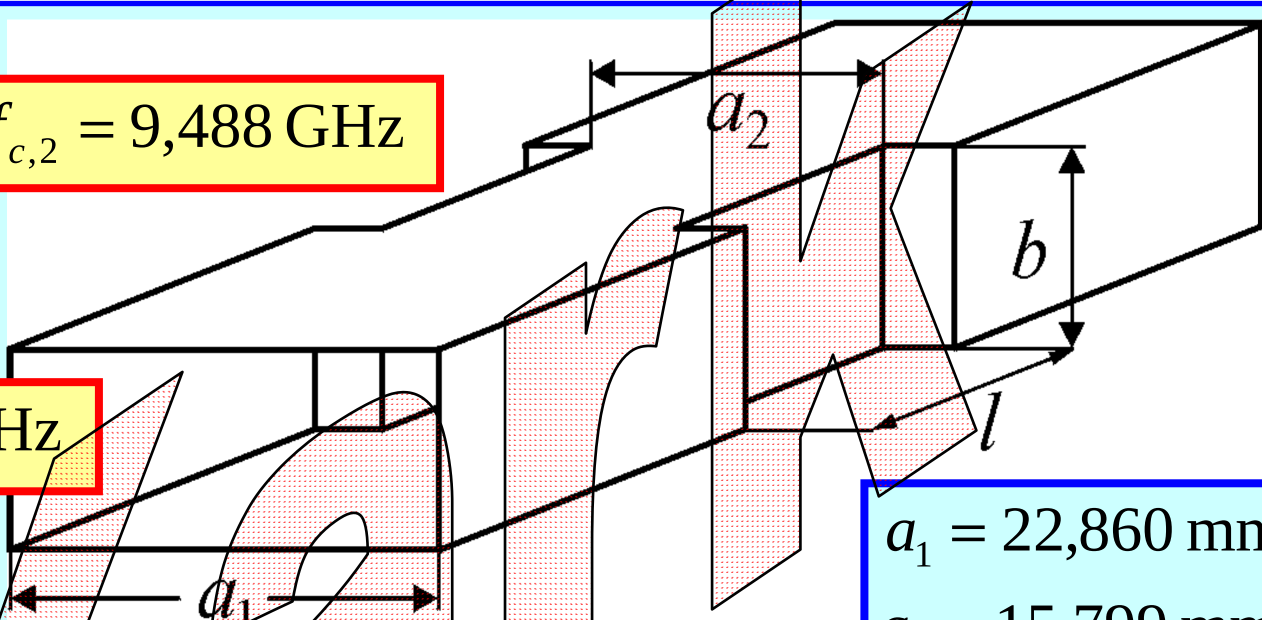
$\tau = 4$ ns



H_{10}-Hohlleiter mit Cutoff-Verengung

Ku-Band: $f_{c,2} = 9{,}488\ \mathrm{GHz}$

X-Band: $f_{c,1} = 6{,}557\ \mathrm{GHz}$



$a_1 = 22{,}860\ \mathrm{mm}$

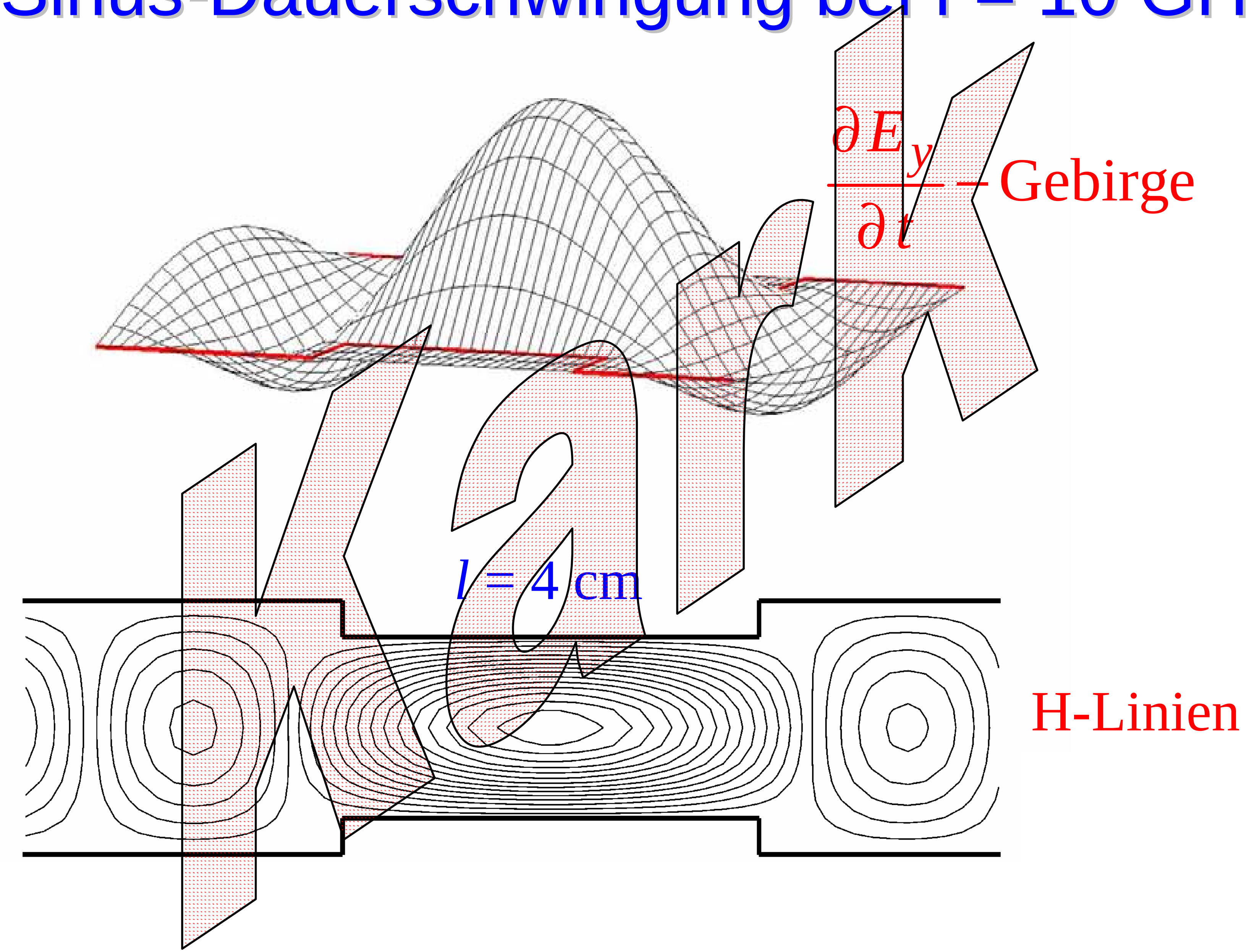
$a_2 = 15{,}799\ \mathrm{mm}$

$b = 10{,}160\ \mathrm{mm}$

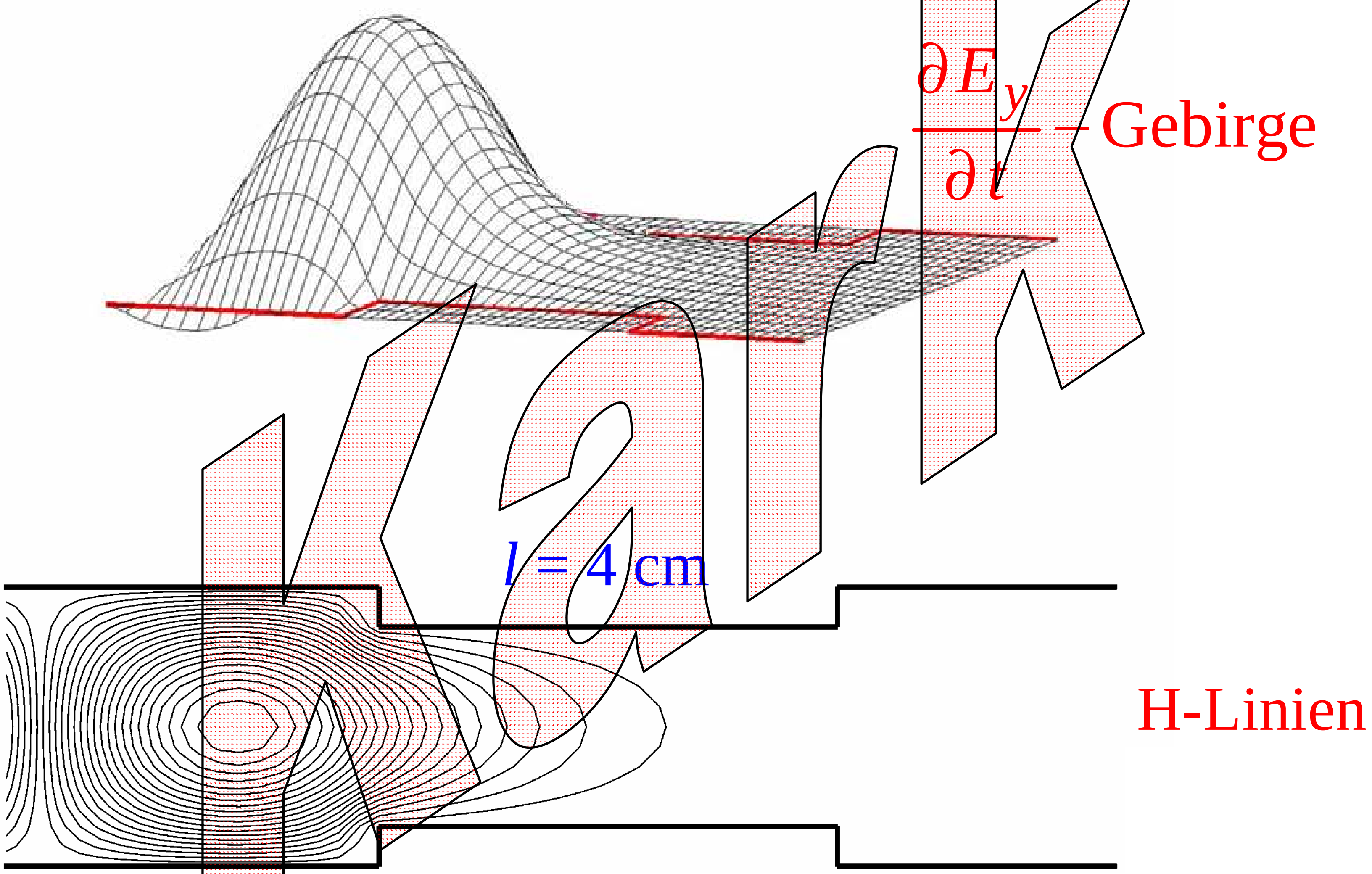
Ungerade H_{m0} – Wellen (m = 1, 3, 5, ...)

$$\underline{A}_z = \sum_{m=1}^{\infty} \cos\frac{m\pi x}{a}\left(\underline{a}_m\, e^{-\underline{\gamma}_m z} + \underline{b}_m\, e^{\underline{\gamma}_m z}\right)$$

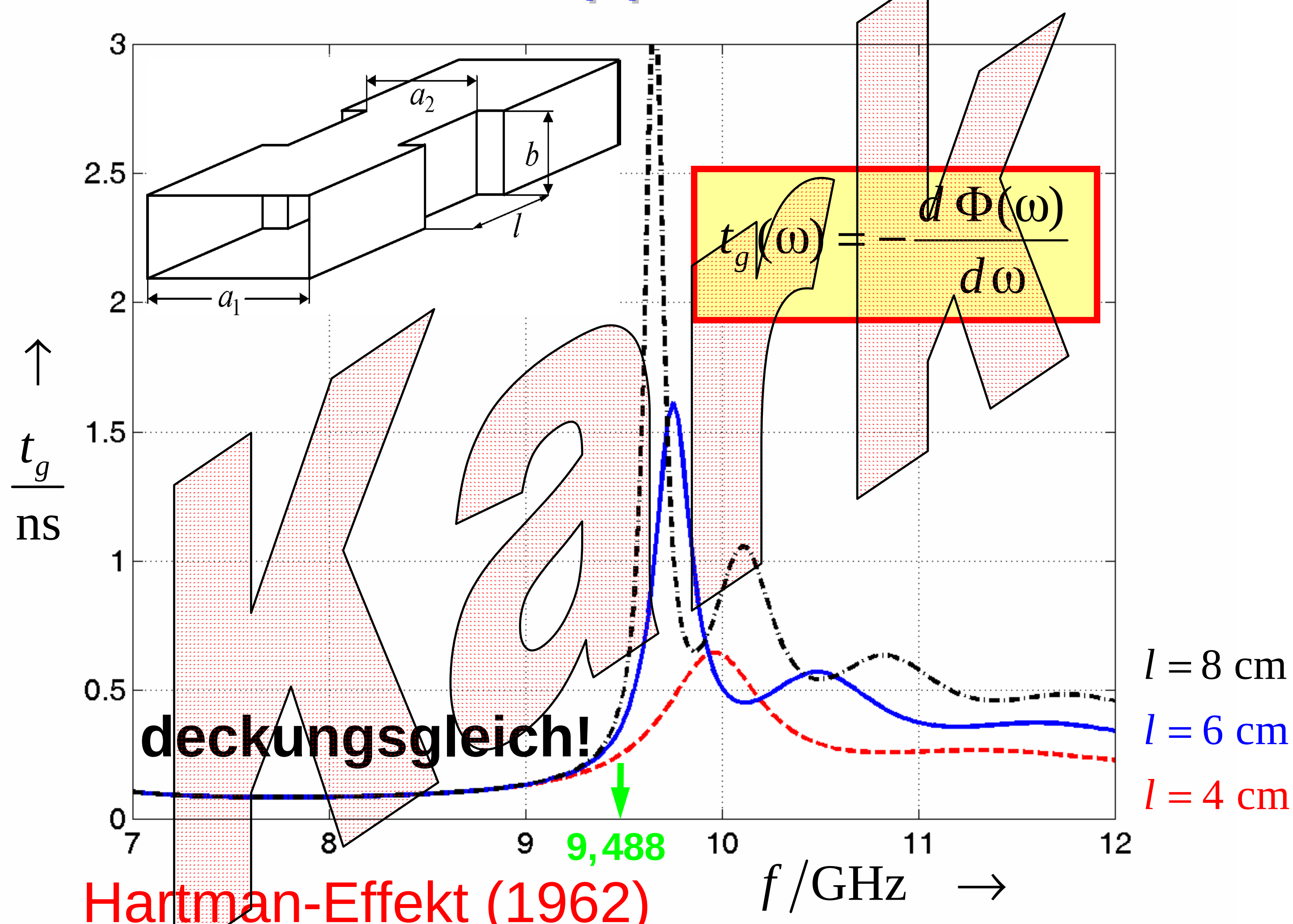
Sinus-Dauerschwingung bei f = 10 GHz



Sinus-Dauerschwingung bei f = 8 GHz

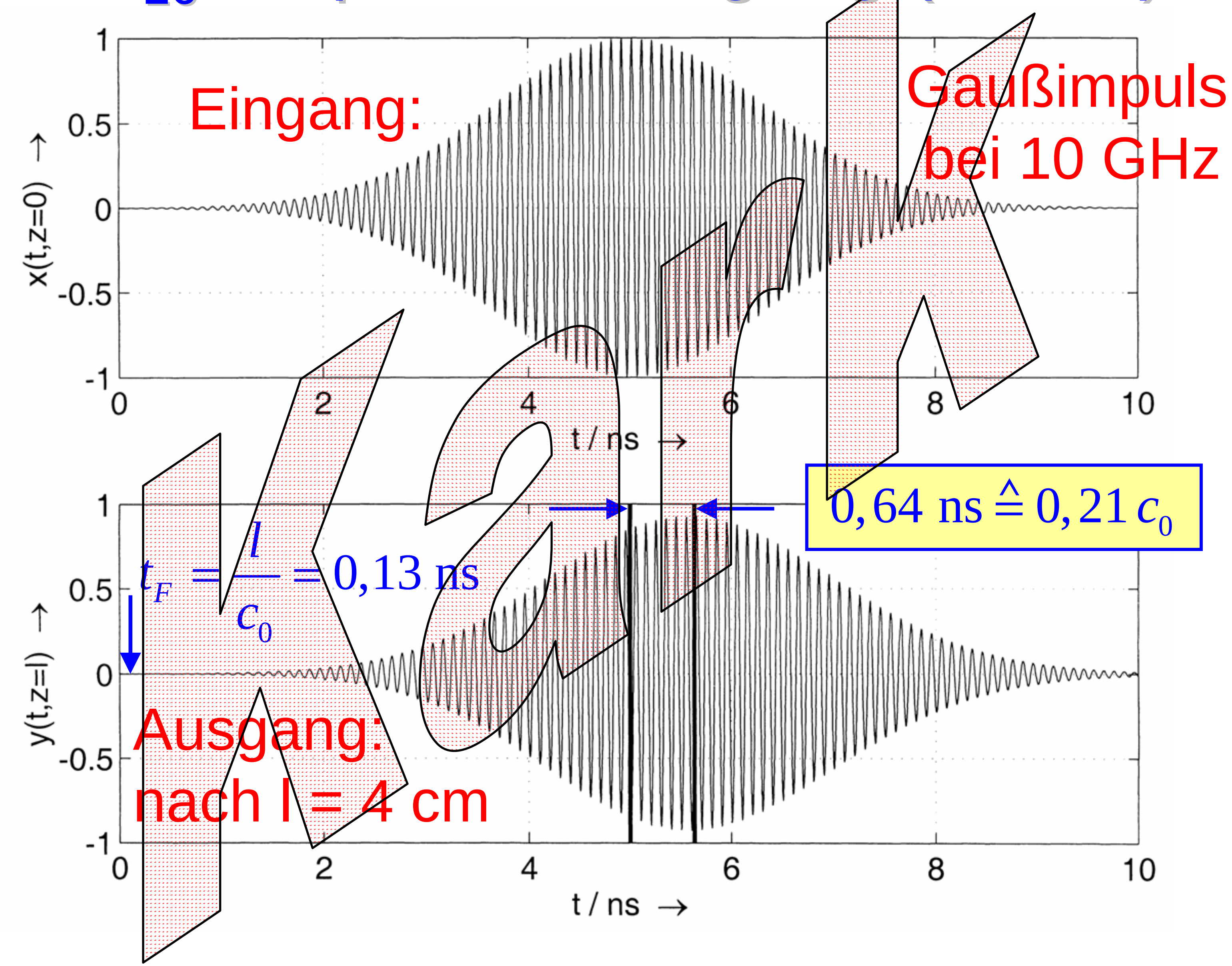


Die Gruppenlaufzeit

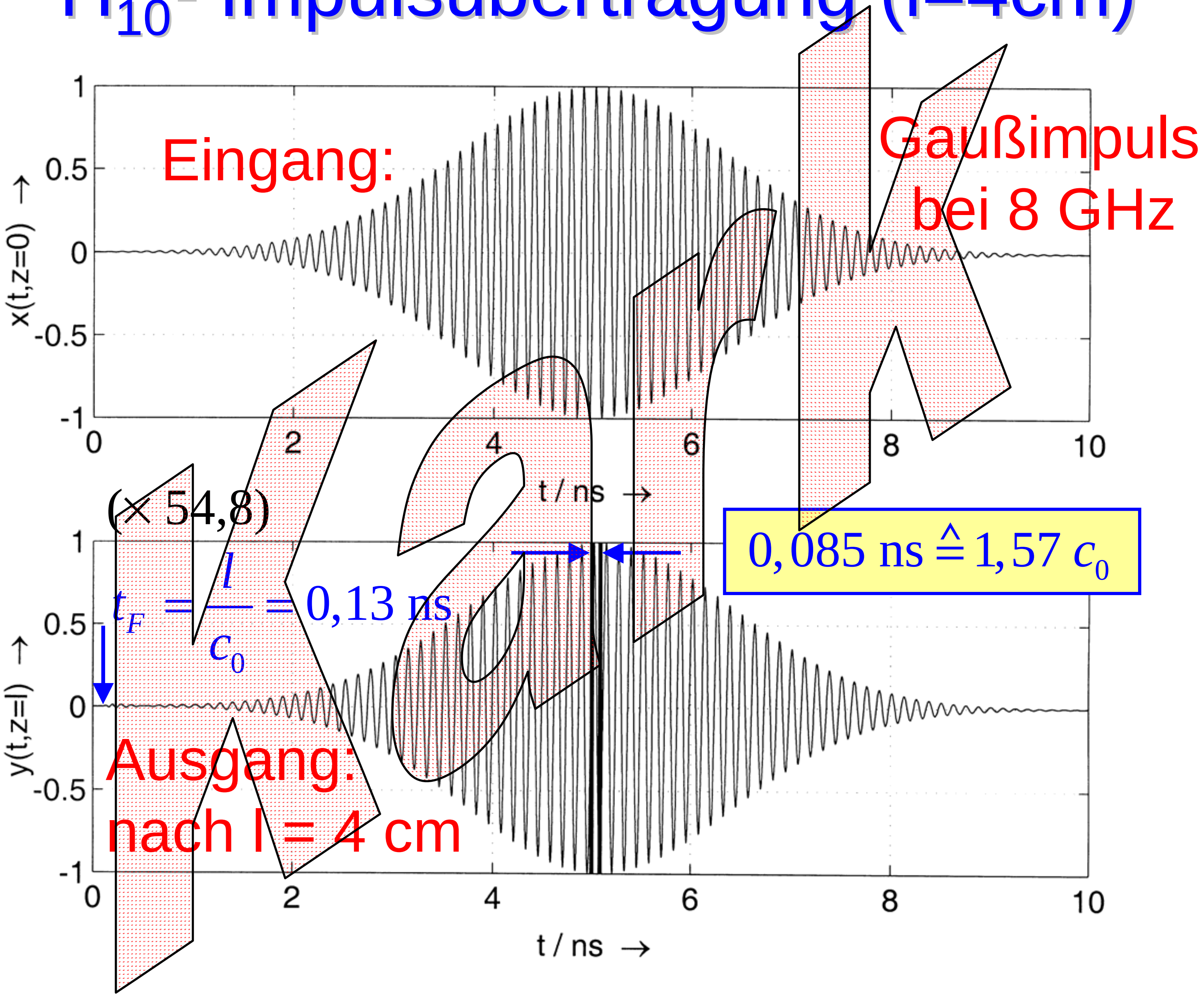


Hartman-Effekt (1962)

H_{10}- Impulsübertragung (l=4cm)



H_{10}- Impulsübertragung (l=4cm)



Superluminales Tunneln – kausal?

Frontlaufzeit (wächst mit Tunnellänge) $\boxed{t_F = l/c_0}$

Gruppenlaufzeit (konstant im Sperrbereich) $\boxed{t_g = 85\ \mathrm{ps}}$

60 dB – Impulsdauer (Impulsmitte bei $\tau/2$) $\boxed{\tau = 10\ \mathrm{ns}}$

Ab welcher Tunnellänge wird $t_F > \dfrac{\tau}{2} + t_g$?

$$\boxed{l > c_0 \left(\frac{\tau}{2} + t_g \right) = 1{,}52\ \mathrm{m}}$$

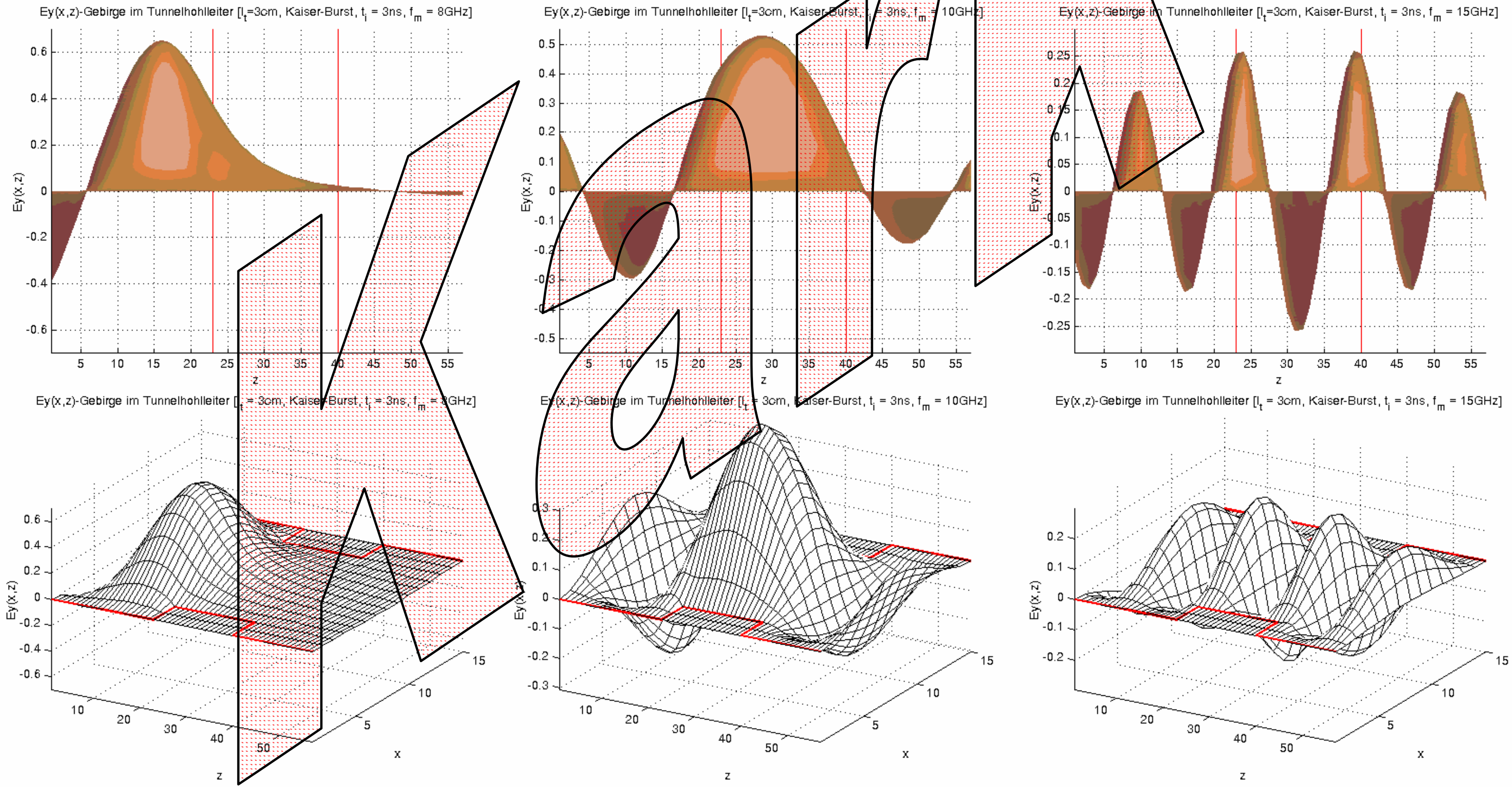
Wie stark ist dann die Dämpfung bei f = 8 GHz ?

mit $\alpha_2 = \sqrt{\omega_{c,2}^2 - \omega^2} \Big/ c_0\,,\ l = 1{,}52\ \mathrm{m}$ wird $\boxed{20 \lg e^{\alpha_2 l} = 1412\ \mathrm{dB}}$

Kaiser-Bessel Burst im Tunnelhohlleiter

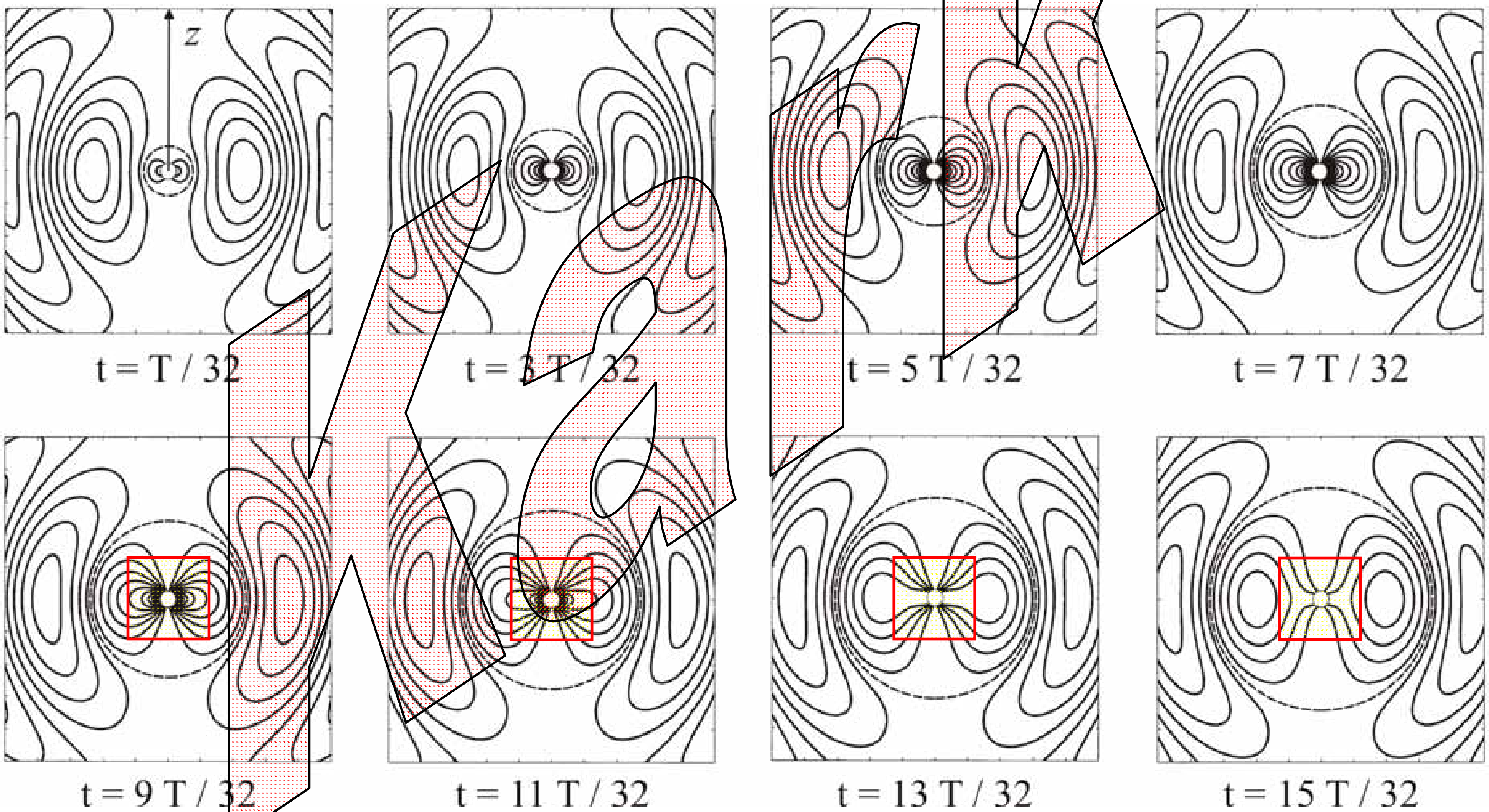
$\tau = 3\ \text{ns}$ und $l = 3\ \text{cm}$

$f_0 = 8\ \text{GHz}$ $f_0 = 10\ \text{GHz}$ $f_0 = 15\ \text{GHz}$



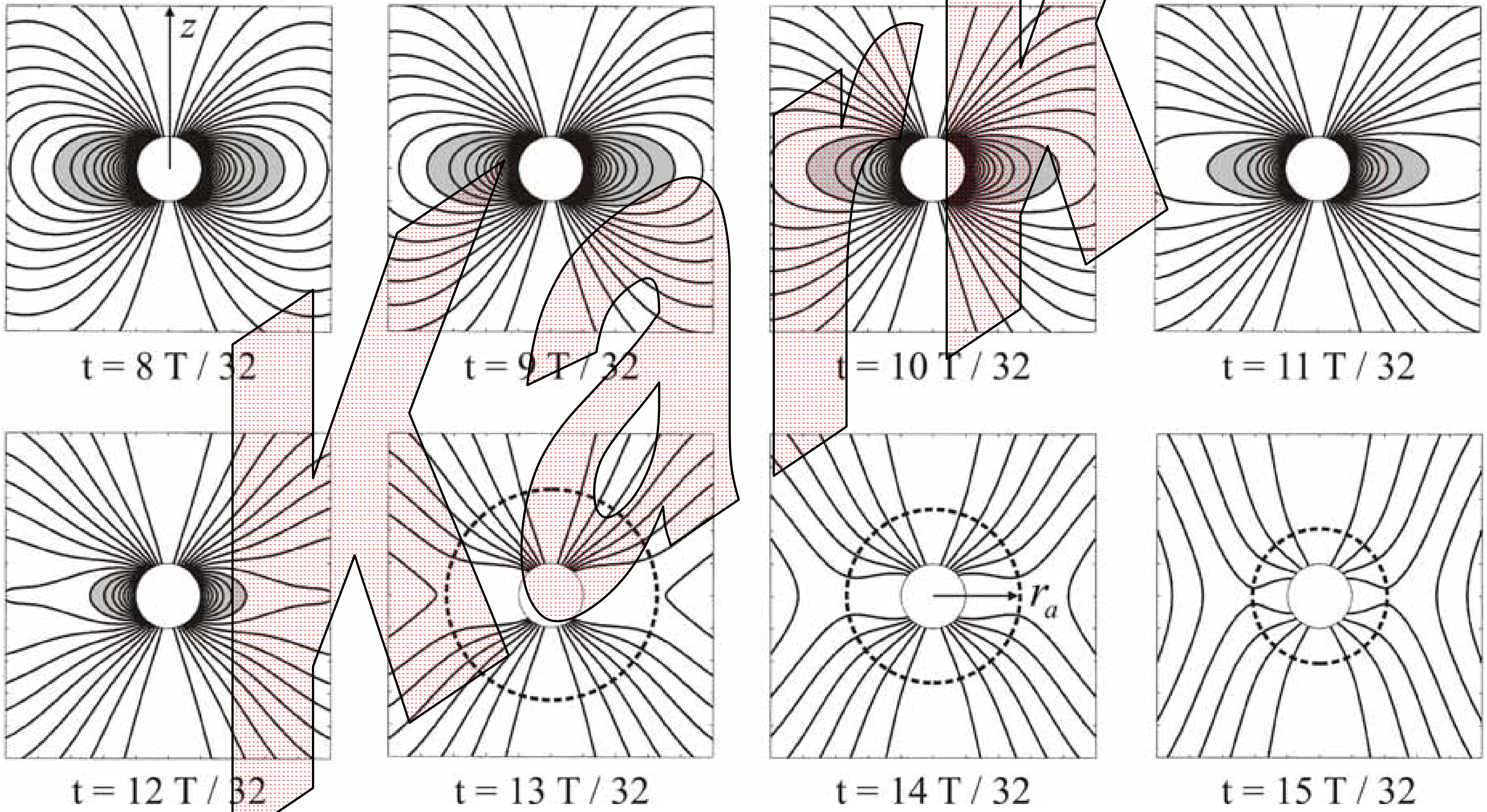
Hertzscher Dipol – elektrische Feldlinien

Darstellungsbereich: $r \le \lambda_0$



Hertzscher Dipol – elektrische Feldlinien

Darstellungsbereich: $r \leq \lambda_0 / 4$



Phasengeschwindigkeit der Dipolstrahlung

$$E_\vartheta = E_0 \frac{\sin\vartheta}{k_0 r} \sqrt{1 - \frac{1}{(k_0 r)^2} + \frac{1}{(k_0 r)^4}}\, \sin\big(\Phi(r,t)\big)$$

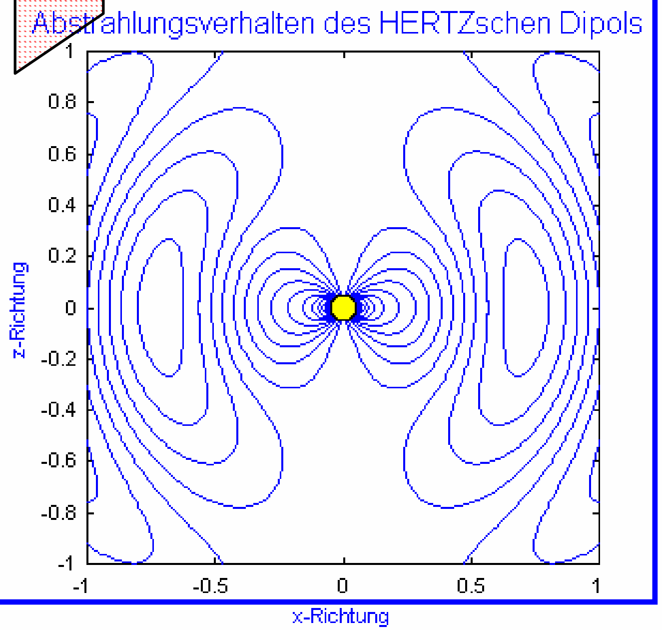
$$\Phi(r,t) = \omega t - k_0 r + \arctan\left(k_0 r - \frac{1}{k_0 r}\right)$$

$$d\Phi = \frac{\partial \Phi}{\partial t} dt + \frac{\partial \Phi}{\partial r} dr = 0$$

$$\mathrm{v}_p = \frac{dr}{dt} = -\frac{\partial\Phi / \partial t}{\partial\Phi / \partial r}$$

$$(\text{Pol bei } k_0 r = \sqrt{2})$$

$$\mathrm{v}_p(r) = c_0 \left(1 - \frac{1 + (k_0 r)^2}{2\,(k_0 r)^2 - (k_0 r)^4}\right) = \frac{\omega}{\beta(r)}$$



Gruppen- und Energiegeschwindigkeit

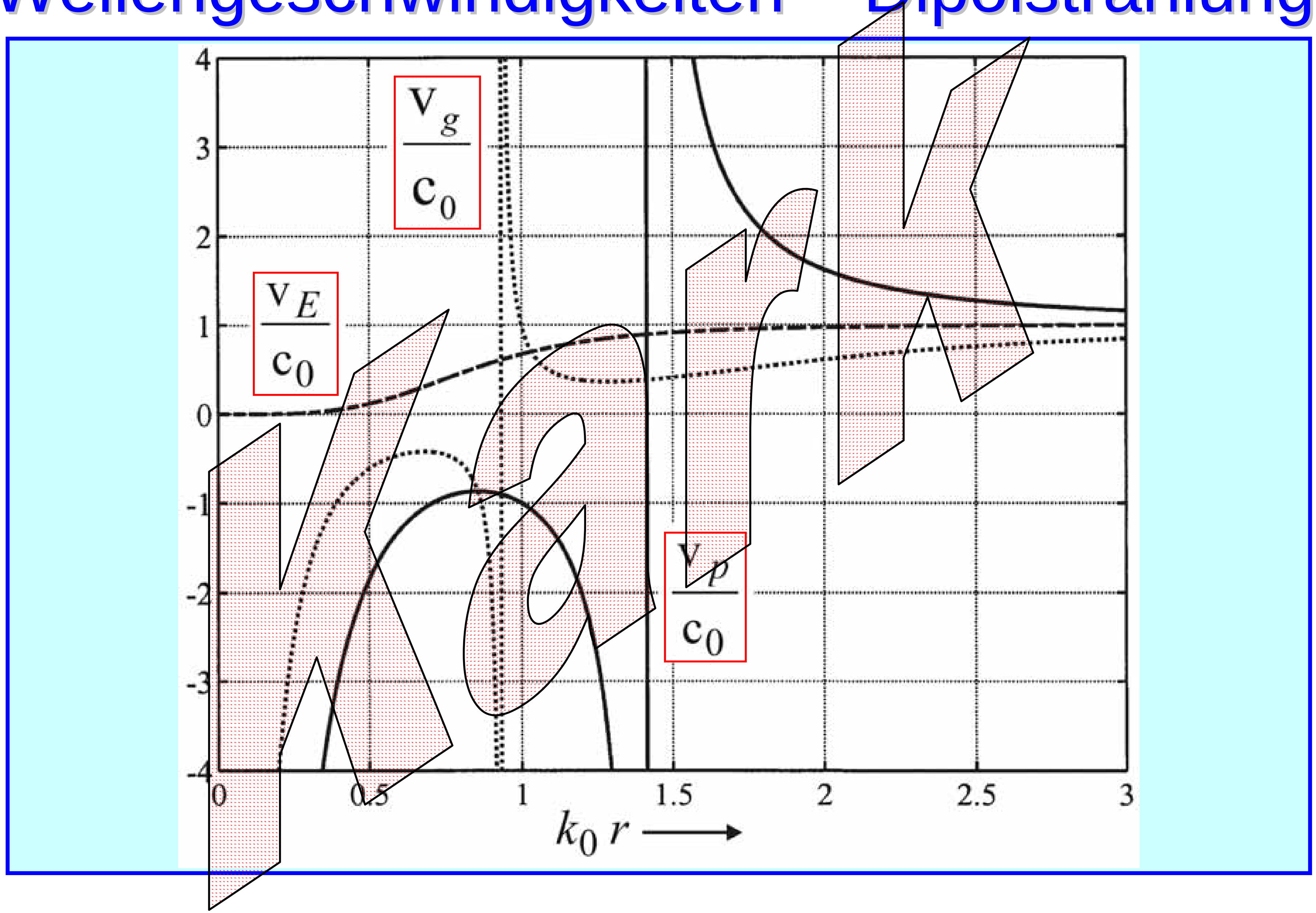
$$\beta(r) = \frac{2\pi}{\lambda(r)} = k_0\left(1 - \frac{1+(k_0\,r)^2}{1-(k_0\,r)^2+(k_0\,r)^4}\right) \xrightarrow[k_0\,r \gg 1]{} k_0 = \frac{2\pi}{\lambda_0}$$

$$\mathrm{v}_g(r) = \frac{\partial\omega}{\partial\beta(r)} = c_0\left(1 + \frac{1+4\,(k_0\,r)^2-4\,(k_0\,r)^4-(k_0\,r)^6}{-6\,(k_0\,r)^2+7\,(k_0\,r)^4-(k_0\,r)^6+(k_0\,r)^8}\right)$$

(Pol bei $k_0 r \approx 0{,}933$)

$$\mathrm{v}_E(r) = c_0\,\frac{2\,\mathrm{Re}\left\{\underline{E}_\vartheta\,Z_0\,\underline{H}_\varphi^*\right\}}{\left|\underline{E}_\vartheta\right|^2 + Z_0^2\left|\underline{H}_\varphi\right|^2} = \frac{c_0}{1+\dfrac{1}{2\,(k_0 r)^4}} \le c_0 \qquad \text{für } \vartheta = \frac{\pi}{2}$$

Wellengeschwindigkeiten – Dipolstrahlung

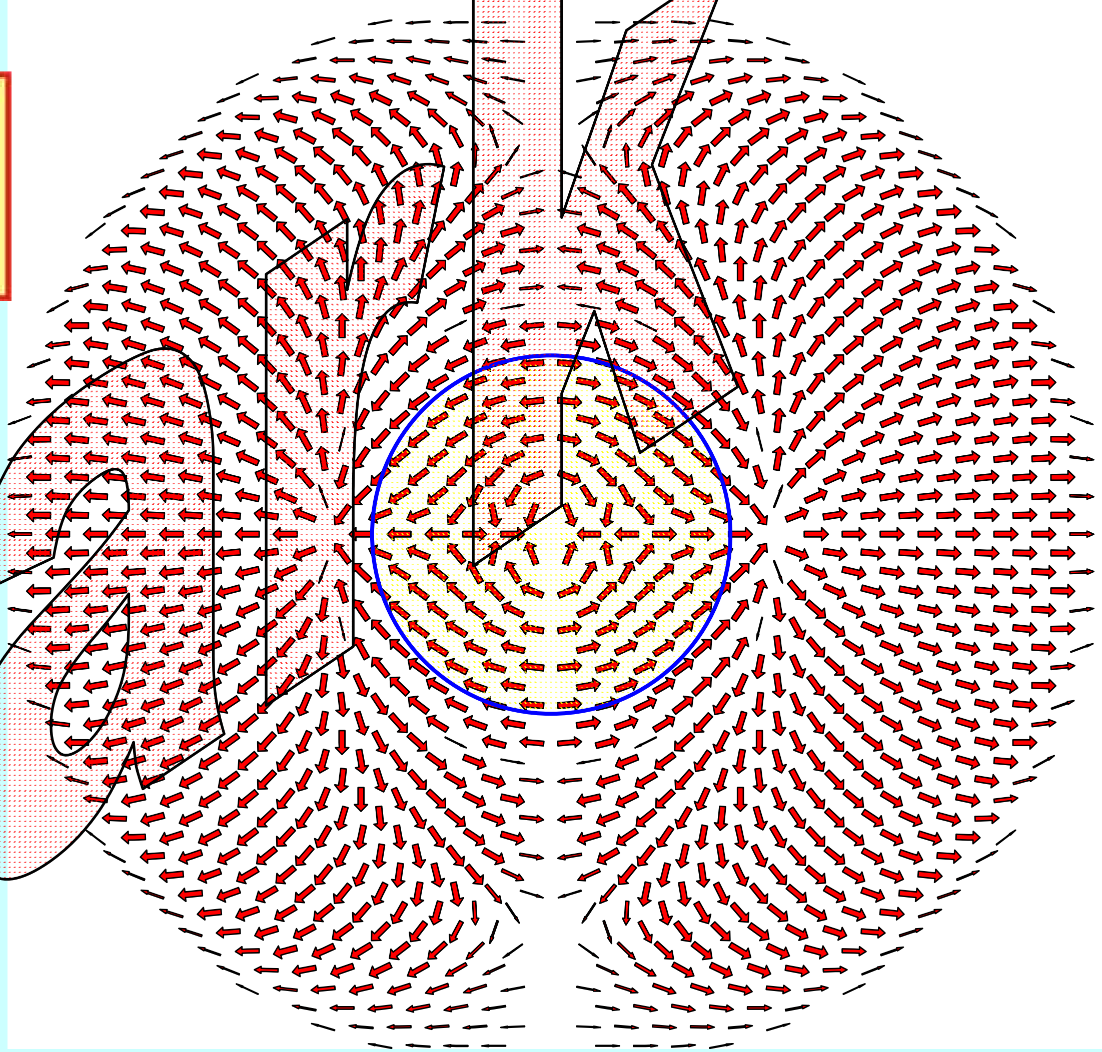


Energiestrom des Hertzschen Dipols

$$\vec{S}(t) = \vec{E}(t) \times \vec{H}(t) =$$
$$= \left(E_r\,\vec{e}_r + E_\vartheta\,\vec{e}_\vartheta\right) \times H_\varphi\,\vec{e}_\varphi$$

radiale und meridionale Komponenten

$$r \le 0{,}9\,\lambda_0$$
$$t = \frac{7}{16}\,T$$



Energiestrom des Hertzschen Dipols

$$r \le 0{,}3\,\lambda_0$$

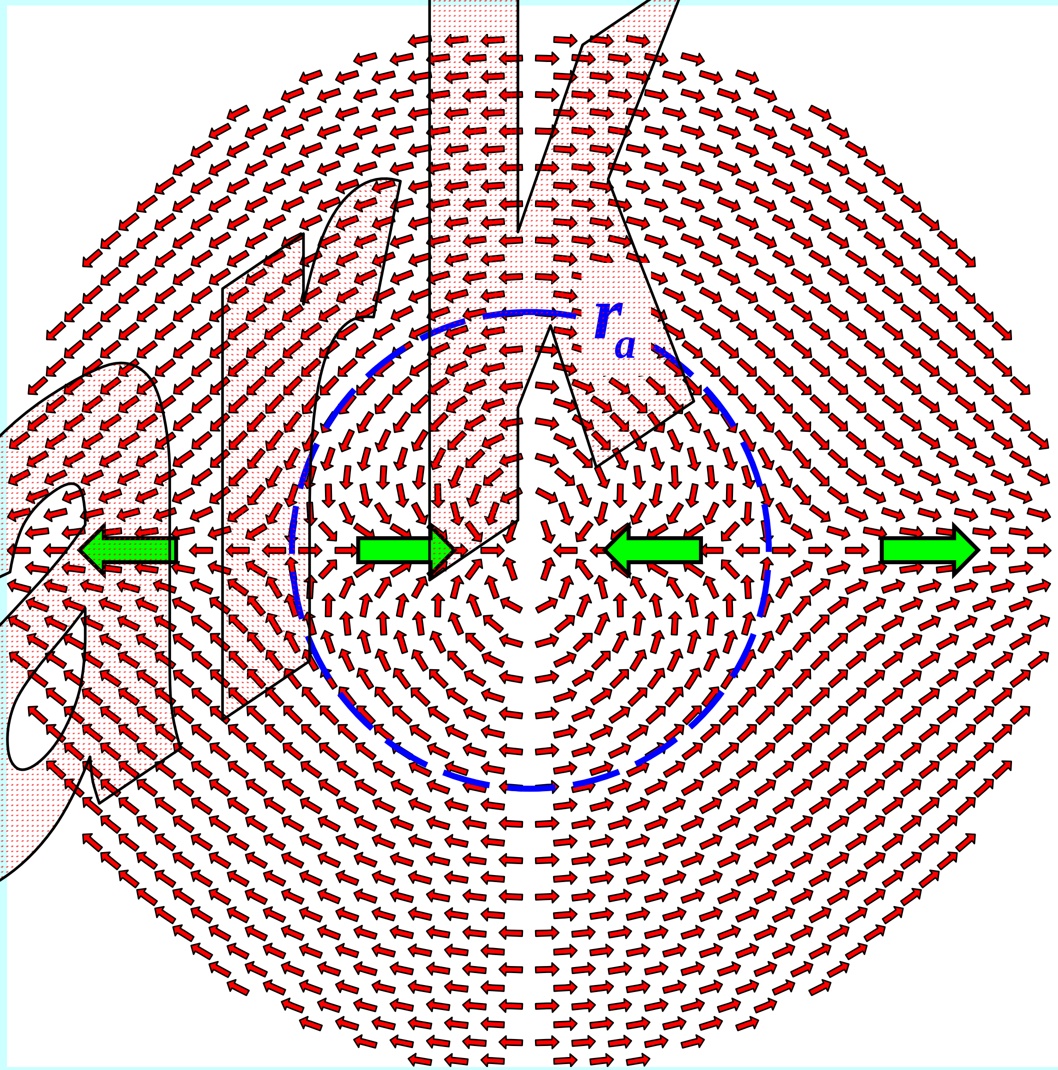
$$t = \frac{7}{16}\,T$$

Rückfluss der Energie für

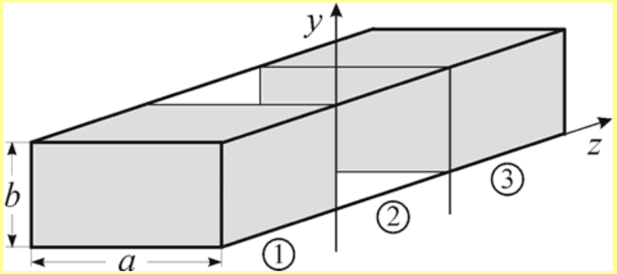
$$\frac{6}{16}\,T < t < \frac{8}{16}\,T$$

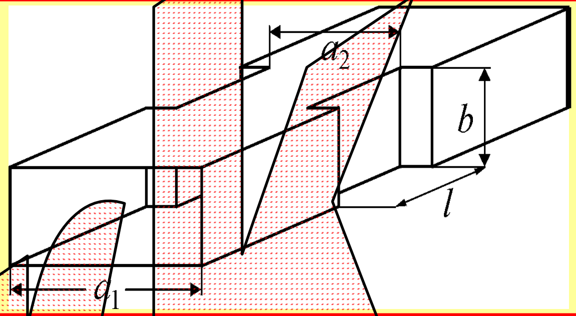
Abschnürradius

$$\sqrt{2} \ge k_0\, r_a \ge 0$$



Zusammenfassung





- Evaneszente Moden in Hohlleitern
- Superluminales Impulsmaximum ($v_g > c_0$)
- Filter mit negativer Gruppenlaufzeit ($t_g < 0$)
- Antennen-Nahfelder ($-\infty < v_{g,p} < \infty$)
- **Kausale** Signalübertragung: $v_F = c_0$ und $v_E \leq c_0$
- Erklärung durch **Maxwellsche Theorie**
- Kein Spielraum für „neue“ Physik!